Aristotle then completes this consideration by indicating where the exercise of art is most clearly like the action of nature. For, he says, this likeness is most clear "when someone cures himself." The difference, of course, as Aristotle had pointed out in the first chapter of this book, is that it is accidental that the principle of healing—the art of medicine—be in the one being healed, while it is essential that the principle which is nature be in the thing moving or developing, as (for example) the principle of growth must be in the growing thing.

CUTTING THE INFINITE DOWN TO SIZE

Marcus R. Berquist

Aristotle defines the *continuous* in two ways. In *Categories*, the Continuous is "that whose parts have a common boundary." In *Physics*, it is "that which is divisible in infinitum." These definitions are not opposed, but complementary: the first is by way of composition—it indicates how the many that compose the continuum are *one*; the second is by way of resolution—it indicates how the one continuum is also *many*.

In both definitions, the *indivisible* ("that which has no parts") is included, at least implicitly. For the common boundary of the parts of a continuum is indivisible, and the division of the continuum is also effected by the same indivisible. In these accounts, the indivisible is never regarded as a *part*; it is that by which the parts are joined, and that by which each part is limited. Thus, the continuum does not consist of indivisibles, nor is it divided into them. A line does not consist of points, nor a motion of moments, nor time of instants.

Now the division of the continuum necessarily involves number: a magnitude is divided into a number of parts, each of which is *one*. But such a part is not perfectly a unit, since it is in turn divisible into many. Because of this, and because of the reasonable premise that the divisible pre-supposes the indivisible, one is inclined to think that there must be an *ultimate* division of the continuum into parts that are absolutely indivisible. Accordingly, the continuum (it seems) must ultimately consist of indivisible units; the line, in the first case, must consist of points, for a divisible consists of the parts into which it is divided. Although this supposition is not consistent with one's prior conception of the continuum, according to which it is only divided into divisibles, it may nevertheless seem to be necessary. Here is Galileo's account of the matter, in his *Two New Sciences*:

> . . . the line, and every continuum, being divisible into ever-divisibles, I do not see how to escape their composition from infinitely many indivisibles; for division and subdivision that can be carried on forever assumes that the parts are infinitely many. Otherwise the subdivision would come to an end. And the existence of infinitely many parts has as a consequence their being unquantifiable, since infinitely many quantified [parts] make up an infinite extension. And thus we have the continuum composed of infinitely many indivisibles.[1]

There is another supposition that tends to the same conclusion. There seems to be a tendency among philosophers to regard the potential as nothing other than a *hidden* actual. Though this supposition is seldom stated as such, it is implicit in many of their opinions. For example, Anaxagoras maintains that whatever comes to be is already actually contained in the material beforehand, but imperceptible because of its smallness. ("Because of the weakness of our senses, we cannot see the truth.") Likewise, we are inclined to think that all the points that might be designated on a line already actually exist there, though one can only attend to them one by one, and therefore never take note *distinctly* of every one of them. Although this is not the same as to suppose that a line is composed of points, in what follows we shall argue that it leads to that conclusion, as well as to other impossible consequences.

Accordingly, we shall not concern ourselves here with the opinion that a line is composed of points. That opinion has been considered and refuted by Aristotle in Book VI of *Physics*. It should be noted in passing, however, that those who regard the continuum as so constituted (as do most of the modern philosophers and mathematicians I have consulted) do not at-

[1] Drake translation, p. 42.

tend to Aristotle's arguments. Thus, Bertrand Russell, for example, defines a line as an *ordered* series of points; and though he recognizes that one point cannot be next to another, he does not explain how points are distinguished from (i.e. are *external to*) one another. Yet distinction is pre-supposed to order. But to discuss this in detail belongs to another consideration. Here we shall examine the opinion that every possible point on a line is also actual.

Let us first consider the points that mark the successive bisections of a finite line. The first point bisects the whole, the second, the remainder, the third, the next remainder, and so on. The parts are, in order, the half, the quarter, the eighth of the whole, and so on, each part being the half of the part before. Now on the assumption that every possible point on the line is actual, all these points constitute an actually infinite multitude. And since every point is the term of a finite segment of the line, these segments are also infinite in multitude. So within the limits of a given finite line, there are infinitely many extended magnitudes, arranged in succession. But it is a common premise that an infinite multitude of finite lines constitutes an infinite magnitude. Therefore, the given line contains an infinite magnitude, and must accordingly be both finite and infinite.

It may be objected that the successive magnitudes get smaller and smaller, and that there is no minimum among them, so that they need not add up to the whole finite line. This objection is not to the point. It matters not to the premise how long or short the component magnitudes are; it is sufficient that each one of them be finite. Accordingly, whatever the variations in size, they must constitute an infinite extension.

Again, if any magnitude consists of many parts actually present within it, one magnitude cannot be greater than another unless it consist of a greater multitude of parts, or a multitude of greater parts. But on the assumption that every possible point on a line is also actual, one cannot suppose that there is a greater multitude of points on the longer line than on the shorter, for it can easily be shown that there is a one-to-one correspondence between the points on any two lines. Accordingly, there is also a one-to-one correspondence between the parts which those points determine. Thus, the longer line does not consist of a greater multitude of parts. (One-to-one correspondence is the criterion for equality of multitudes, comparable to co-incidence in magnitudes.) Neither can it be assumed that the segments are longer in one case than in the other, for if they were, the longer segments would not have been divided *through and through*. There would be a further division which is only potential, contrary to the assumption. Thus, all magnitudes would consist of equal multitudes of identical parts, and one cannot be greater than the other.

Furthermore, every multitude consists of units, that is, must be many *ones*. But on the assumption that every possible division is actual, any finite part of the line must consist of many magnitudes actually present within it, each of which is extended and actually divided into yet other magnitudes that are likewise divided. So long, then, as all these multitudes consist of other multitudes, one cannot designate any unit other than *one multitude*; one cannot designate the units from which all these multitudes are composed. The units of which multitude *as such* must consist are nowhere to be found among these divisions. It is as if one were to identify the number ten with the unit, because each of the tens that add up to one hundred is *one ten*. Accordingly, since a multitude must ultimately consist of ones, and not of other multitudes, no unit can be posited other than the point. Thus, the division of magnitude becomes like that of number. Just as numbers can be divided into other numbers, but must ultimately be divided into units, which are not numbers, so must magnitudes be ultimately divided into points, which are not magnitudes. Therefore, a line must consist of points. It is not surprising, then, that Galileo, who regards every point on a line as fully

actual, should also compose a line from indivisibles, and motion from indivisible moments.

The foregoing difficulties arise even when one is considering magnitude in the abstract. But since natural things have magnitude, and their motions depend upon magnitude, one should consider the consequences for natural science of the supposition in question.

The natural as such is defined by motion and change, and all other kinds of change pre-suppose change of place. Thus, if there is no such thing as local motion, there is nothing natural and no natural science. (The early Greek naturalists in effect denied all change except change of place; they seem to have realized that to deny that sort of change would be to deny the natural altogether, as Parmenides did.) Further, because such motion pre-supposes the continuity of magnitude, one must ask whether motion would even be possible, if every possible division of the continuum is actual.

The consequences of this supposition, as they concern motion, are evident. In the first place, to traverse any magnitude whatever, a mobile would have to traverse the infinite, since the finite parts of the continuum (on this supposition) are infinite in multitude. But this is impossible, for one cannot take the all of the successive unless there is a last of it, and there is no last of an infinite multitude or magnitude. And even if one could traverse such a magnitude, one would need an infinite time to do so.

Furthermore, since, on this same supposition, one magnitude cannot be longer than another, there would be no such thing as speed, for the swifter is what traverses a greater magnitude in the same time.

Moreover, since the divisions of motion and time follow from and correspond to the divisions of the magnitude traversed, and the latter (by this supposition) consists of a multitude of actually distinct parts, bounded by actually distinct points, the motion over a magnitude must consist of actually distinct motions, bounded by actually distinct *moments*. Time likewise must consist of actually distinct parts, bounded by actually distinct *instants* (or "nows"). But in this case (as argued above) a magnitude will *ultimately* consist of nothing but points, and motion over it will consist of indivisible *moments*, one after the other, and the time will also consist of indivisible instants. In other words, corresponding to the indivisibles of magnitude, there will be indivisibles of motion and of time. But then, as Aristotle notes, the mobile will *have moved* without ever *moving*, and there will be no such thing as motion, properly speaking, but only the successive occupation of places; there will be no *becoming*, but only *being*.

Yet even this sort of progression from place to place is impossible. For a mobile can cease to be where it is only insofar as it comes to be somewhere else. But this requires that there be a *next* place, which is impossible, if magnitude is composed of indivisibles.

Finally, even a preliminary consideration of this issue, such as this, would be incomplete without a consideration of the arguments which St. Thomas gives against the very possibility of an infinite magnitude or an infinite multitude. As regards the former, here is St. Thomas' argument:

> . . . every natural body has some determinate substantial form. Since, therefore, accidents follow upon the substantial form, it is necessary that determinate accidents follow upon a determinate form, among which [accidents] is quantity. Whence, every natural body has a determinate quantity, both in greatness and in smallness. Whence it is impossible that any natural body be infinite. This is also clear from motion. An infinite body cannot have any natural motion: in a straight line, because nothing is moving naturally with a straight-line motion except when it is outside its natural place, which cannot be for an infinite body, for it would occupy all places, and thus indifferently any place would be its place. And likewise also neither [would it move] with circular motion, because in a circular motion it is necessary that one part of the body be transferred to the place in which another part was. But in a circular body, if it be

> supposed infinite, this is impossible, because two lines extended from the center, however farther they are extended from the center, by so much do they stand apart from one another. If therefore the body were infinite, the lines would stand apart from one another *in infinitum*, and thus one [part] could never arrive at the place of another.
>
> Concerning the mathematical body also there is the same argument. For if we imagine a mathematical body, when it exists in act, it is necessary that we imagine it under some form, because nothing is in act except through its form. Whence, since the form of the quantified, insofar as it is such, is shape, it will be necessary that it have some shape. And thus it will be finite, for a figure is what is comprehended by a term or terms.
>
> . . . although the infinite is not against the notion of magnitude in common, it is necessarily against the notion of any species of quantity, namely, against the notion of a two-foot or three-foot magnitude, or a circular or triangular [magnitude], and the like. But it is not possible that there be in a genus what is in no species [of that genus]. Whence it is not possible that there be any infinite magnitude, since no species of magnitude is infinite.[2]

St. Thomas argues further against the possibility of an infinite multitude:

> . . . every multitude must be in some species of multitude. But the species of multitude are according to the species of numbers. But no species of number is infinite, since any number whatever is a multitude measured by one. Whence it is impossible that there be a multitude infinite in act, either *per se* or *per accidens*.[3]

Modern mathematicians and philosophers are not aware of, or do not consider, arguments such as these, perhaps because their intellectual custom stands in the way. But it is not difficult to see the relevance and force of St. Thomas' arguments

[2] *Summa Theologiae*, Ia, Q. 7, a. 3.
[3] *Summa Theologiae*, Ia, Q. 7, a. 4.

against the supposition of infinite quantities. It is hardly reasonable to posit *kinds* of quantity that can be defined only by the *negation* of what constitutes other kinds. It is as if one were to define a pig as an irrational animal. Nor is it sufficient to say that the infinite *exceeds* all other quantities; that would be like defining the number seven as that which exceeds two, three, four, etc. But a fuller discussion of this is beyond our scope here.

In conclusion, one may wonder about the sources of the difficulties about the infinite which have been discussed above. This would require a lengthy examination, to be sure, since in most cases the good and the true come about in only one way, the bad and the false in many ways, perhaps in infinite ways. One could make a joke, and say that errors about the infinite naturally come about in infinite ways, and he would have some support for his joke in the multitude of books and articles that have detailed "the paradoxes of the infinite." But let us consider at least some of the likely sources of difficulty and misunderstanding on this issue.

In the first place, one does not sufficiently attend to the perceptions and reasoning that lead us to our notion of the infinite, and the meaning of the names we give. One has not seen or imagined an infinite magnitude or multitude; rather, he has realized that there is no greatest or least of the finite. It is a discovery of what *is not* rather than what *is*. Thus, an infinite magnitude or multitude is in no way *given*, to sense, to imagination, or to understanding. One must reason that there are such quantities.

Another cause of difficulty was noted above. Because the potential as such is less intelligible, one tends to identify it with the actual by which it is understood. Thus we speak of *designating* or *marking* the points on a line, as if they were all actually there.

Again, one is inclined to think that the universe is infinite, because when he imagines a boundary, he also imagines something else beyond, some extension, at least. Resolving

to the imagination, then, one regards it as almost self-evident that space is infinite. One forgets that what one must imagine when one is thinking of something need not be an attribute of that thing. For example, in studying geometry, one must imagine the figures, and this requires that they be imagined as colored, but it would be foolish to conclude from this that they *must* be colored, as if color belonged to them *per se*, or even were necessary for them to exist in natural bodies.

Finally, though reference is sometimes made to Aristotle's treatment of this issue in Books Three and Six of *Physics*, mathematicians and philosophers seldom attend to what he says there. Not only do they ignore what he says about the composition of the continuous, as noted above, but also misunderstand what he says about the infinite. In particular, they misunderstand what he means by "potentially infinite."

Some understand "potentially infinite" the way we understand "potentially hot." When we say that the water is potentially hot, we mean that it *can be* hot. Here there is no paradox in saying that the water may actually be hot at some time. Thus, if we understand "potentially infinite" in this way, we can hardly deny that the infinite may also be actual. This is how Galileo, in his *Two New Sciences*, understands the matter:

> . . . the quantified parts in the continuum, whether potentially or actually there, do not make it quantity greater or less. But it is clear that quantified parts actually contained in their whole, if they are infinitely many, make it of infinite magnitude; whence infinitely many quantified parts cannot be contained even potentially except in an infinite magnitude. Thus in the finite, infinitely many quantified cannot be contained either actually or potentially.[4]

But this a misunderstanding of the phrase, as Aristotle makes clear:

> One must not, however, take "being in potency" as [meaning that, e.g.,] if this statue is able to be, this statue also will

[4] Drake translation, p. 43.

> be, and so also the infinite is what will be in act, but [rather] since being is in many ways, as a day or the games exist by different [parts] always coming to be, so too does the infinite.[5]

In numbers and in the divisions of a line, the possibilities are infinite. But this does not mean that the infinite is one of the possibilities. A man contemplating marriage may consider three possible wives, but this does not mean that having three wives is among his possibilities. The dative in Greek and the adverb in English *specify* the adjective in some way, but they do not determine *in what way* the adjective is to be specified. In the present case, to say that there is a potential infinite does not mean that there can be an infinite number, or even an infinite number of possibilities (as if one could number the possibilities). Rather, it means that there is no greatest of the possible numbers (or multitudes), even though every number is finite. This is comparable to the infinitely small in magnitude. Though some have thought that there were such magnitudes ("infinitesimals"), most mathematicians now recognize that there are none, and that "infinitely small" means only that there is no smallest possible.

The need to rightly distinguish the meanings here may be illustrated by the similar case of the phrase "infinite power." When we say that an agent has infinite power, we might mean that he is capable of an infinite effect. Or we could mean that there is no greatest of his possible effects, though every one of them is finite. The potentially infinite in magnitude and multitude is like the second of these.

[5] *Physics* III, 206a19–23.

The Axiomatic Character of the Principle that the Common Good Is Preferable to the Private Good

John Francis Nieto

1. The intention of the following remarks is to manifest as distinctly as possible that the principle that the common good is more desirable than the private good is an axiom, that is, an indemonstrable principle that is not confined to one science but is found in each according to the manner appropriate to that science. This is not to deny that the principle is usually contracted to the science of politics, where it enjoys the sort of preeminence that another axiom, the whole is greater than the part, enjoys in mathematics. And much of the consideration that follows will attend to the principle according to the force that it has in political science. Nonetheless, the principal intention is to manifest that it is an axiom and should be understood even in that science as an axiom.

2. Of course one cannot demonstrate an axiom nor can one demonstrate that some axiom is an axiom. Rather one must manifest the truth of the axiom in such a way that manifests that this truth is the kind of truth possessed by axioms. Such truth is not only known *per se*, but it is intelligible through our concept of being and the concepts that are convertible with that of being. I therefore intend to manifest three things: that

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