frequency of its vibration will be exactly three times that of the other. The ratio in either case is 1:3, with the only difference that if the numbers stand for frequencies the greater number represents the higher tone, while if they stand for string lengths it is the other way around: the greater number indicates the longer string, the lower tone. We shall deal with the problem of tone and number more extensively in a later context.

Selection of Tones

So much about tones as such. What about their order as exhibited in the hymn tune?

We observe what goes on. At first there is an ascending sequence that touches three different tones. After this, the direction is reversed. The 4th and 5th tones, which are touched on the way down, are the same as the 2nd and 1st, respectively. (It is by no means self-evident that this should be so.)

The 6th tone is new; so is the 7th. The 8th is the same as the 3rd, and for the rest of the tune no other new tones will appear.

Theoretically the number of possible tones within the limits of our perceptive faculty is infinite. From an infinite number of possible tones our hymn tune has selected precisely five to serve as its tonal material.

The way of this hymn tune is the way of music in general. From a theoretical infinity of possible choices music selects a strictly limited number of tones to serve as the material for all its structures. The validity of this sweeping generalization will appear as we proceed.

As it stands here, this statement seems open to immediate challenge. Suppose we sing the hymn tune twice, the second time beginning a little higher or a little lower. The tones will then all be different; yet it will still be the same old hymn tune. Strictly speaking it cannot be true that this tune selects five

tones that make up its material, as it is possible to build the same tune from a different set of five tones. It is not at all easy to understand how this can happen: how the whole—the tune—can be the same although all of its parts have been changed. (A new psychological theory, the so-called *Gestalt* theory, has sprung precisely from the observation of this musical phenomenon.) There seems only one way to explain it: What matters in a tune is not the tones as such but the relations between tone and tone. Whatever the tones are, as long as these relations remain the same, the tune itself will be the same. It was therefore not quite correct to say that a tune selects a number of tones; what it actually selects are tone relations. Its order is an order of tone relations rather than of tones.

The same is true of music in general. When we say that music selects a definite set of tones for its material, this selection does not fix the exact pitch of the tones; but it does fix the relations between the tones. It selects not tones but an *order* of tones.

The term "music" as used in the last paragraph stands for a concept, an abstract thing. Concretely, "music" exists no more than "language." What actually exists is not "language" but specific languages, not "music" but specific musics. By and large, language boundaries follow national boundaries, music boundaries follow the boundaries of civilizations. We have Western music, Chinese music, East Indian music, Islamic music, and so on.

The basic feature that distinguishes one music from another is the selection of its tonal material. Each music makes its own selection of tones or, to be precise, of tone relations; it selects a specific order of tones. This selection constitutes the tonal system of that particular music. Some civilizations have more than one tonal system, more than one kind of music. Western music for the last 2,500 years has been thriving on one selection, one tonal system. Lately (that is, during the last 100 years or so) a new selection is gradually emerging. Whether this indicates growth or decay is still a matter for debate. In any event, it is an unavoidable development, and it unavoidably creates strife, perplexity, and dismay.

The barriers between music and music are far more impassable than language barriers. We can translate from any language into any other language; yet the mere idea of translating, say, Chinese music into the Western tonal idiom is obvious nonsense. We can take a course and learn the Chinese language; but we must actually live with Chinese music and to a certain extent become Chinese if we want to understand the Chinese tonal language. The favorite quotation about music as the universal language of mankind only betrays a naive tendency on our part to think of ourselves, the representatives of Western civilization, as representing all of mankind.

Dynamic Quality

In talking of our hymn we have used the words *tune, melody.* What is a tune, or melody? What distinguishes a melody from a random succession of tones? Is it the fact of selection, of order? Is it correct to define melody as a succession of tones belonging to a certain order?

There is no composition of Western music, no matter for what instruments or voices it has been written, that cannot be played on a piano. In the white and black keys of the piano keyboard we have at our disposal the full set of tones from which the musical compositions of our civilization have been constructed. Yet a cat walking across a keyboard does not necessarily produce a melody. The fact that the successively sounding tones all belong to the tonal order of our music does not guarantee that the tone sequence is a melody.

We experiment with our hymn tune. We play or sing the tune from the beginning, but instead of ending [see "Alleluia"] we try this: fig. 1 |.

This reminds us of our first experiment with the Beethoven Bagatelle. The normal reaction of the listener will be: No! This is no ending. It is incomplete. It sounds like an unfinished sentence. It leaves us with an acute expectancy of something more to come, something that would complete the statement.

This time we look closer to see what makes us react that way. It is a certain quality or property of the tone "ti", the last tone of the melody in the changed version: something we sense in the tone, a state of unrest, a tension, an urge, almost a will to move on, as if a force were acting on the tone and pulling it in a certain direction. We shall call this quality of the tone its *dynamic quality.*

If we compare this to the behavior of our original closing tone, la as it appeared at the end of the tune, we shall notice a marked difference. While the other tone, ti, seemed to point towards something beyond itself, this one, la, seems exactly the thing towards which the other was pointing. While the first created expectancy, this one satisfies it; while the first appeared in a state of unrest, of tension, lacking balance, this one appears fully at rest, self-satisfied, perfectly balanced. It is this difference of dynamic quality that makes la fit, ti unfit, to serve as the closing tone of our tune.

The dynamic quality of a tone is part of the immediate tone sensation. We *hear* it just as we hear pitch or tone color—but not under all circumstances. A tone must belong to a musical context in order to have dynamic quality. Within a musical context no tone will be without its proper dynamic quality. Outside the musical context, however—for instance, in the laboratory—tones have no dynamic qualities. Thus, the dynamic quality of a tone is its musical quality proper. It distinguishes the musical from the physical phenomenon.

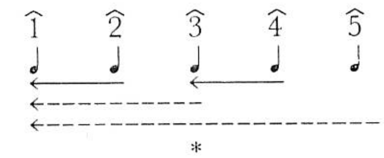
$\hat{1}$ $\hat{2}$ $\hat{3}$ $\hat{4}$ $\hat{5}$

We proceed to experiment with the dynamic qualities of the five tones appearing in our tune. To do this, we bring each tone in a position where it has a chance to reveal its dynamic quality clearly and fully to the ear. Best suited to this purpose is always the position at the end of a melodic phrase. We change the ending of the tune accordingly, trying this fig. 2, or this fig. 3, or this fig. 4. (It is essential in these experiments that the tune always be played from the beginning; if the whole is not heard, the result may be inconclusive.) What do we hear? What does the tone at the end tell us about its dynamic state, its tendency?

What we hear is this: The tone re is very much like ti in its lack of balance, its tendency to move on, except that the tone towards which it is leaning and where it wants to move to is not la but do. (This we find out by comparing the two endings re-do and re-la.) The tone do shows a sort of mixed state. Compared to ti or re it appears better balanced, more at rest; yet if we compare it to la, we immediately realize that its balance is by no means perfect; there is, so to speak, an inner tension left in the tone, and if we try other tones to find out where this tension is directed, la will again emerge in this preferred function. The same description applies to the tone mi, although the character of its tension, of its pointing towards la, is clearly different from that of the tone do; the difference is immediately manifest to the ear, but it cannot be described in words.

Thus, each of the five tones investigated so far has its own distinctive dynamic quality. We shall identify the

dynamic qualities of the tones of this series by the symbols $\hat{1}$, $\hat{2}$, $\hat{3}$, $\hat{4}$, $\hat{5}$. They can be classified according to dynamic state: perfectly balanced, $\hat{1}$; comparatively balanced, $\hat{3}$ and $\hat{5}$; unbalanced, $\hat{2}$ and $\hat{4}$. The following diagram is an attempt to translate the results of these observations into graphic symbols:



The Oscilloscope

We know now a little more about the order of tones and about melody. Tonal order in music is a dynamic order; a succession of tones is a melody when the tones exhibit definite dynamic qualities and, in their succession, reveal a dynamic order.

In our hymn the dynamic qualities $\hat{1}$, $\hat{2}$, $\hat{3}$, $\hat{4}$, and $\hat{5}$ were associated with the pitches la, ti, do, re, mi. They are not tied to these pitches, though; they are not attributes of them. To show this, we sing or play the tune, beginning on sol instead of la. It appears that the tone la, which in the former context exhibited the perfect balance of $\hat{1}$, now conveys the opposite dynamic state of $\hat{2}$; that the tone re, which before carried the quality $\hat{4}$, now carries the quality $\hat{5}$, while $\hat{4}$ has been moved to the tone do, the former carrier of $\hat{3}$—and so on. These experiments can be multiplied. The conclusion: a given pitch can assume any dynamic quality, and a given dynamic quality can associate itself with any pitch.

The dynamic tone qualities are not a new discovery turned up in the course of this study. They form the basis of most of our conventional ear training; but they are never explicitly mentioned, nor are they recognized for what they are: namely, not just another acoustical property but *the* musical property of tones, the property that makes music possible.

With the help of an ingenious instrument, the oscilloscope, tones can be made visible. If a tone is sounded in front of the instrument, a picture of a wave line lights up on a little screen. To every acoustical property of the tone there corresponds a certain feature of the wave line: to pitch, the horizontal distance from crest to crest; to loudness, the vertical distance from crest to trough; to tone color, certain characteristics in the profile of the wave. An experienced observer can read from the picture everything pertaining to the tone—with one exception. The one thing he cannot read from it is the dynamic quality of the tone, its musical quality. Nothing in the picture corresponds to it. The wave line will be exactly the same whether the dynamic quality of a given tone be $\hat{1}$, or $\hat{2}$, or any other. The slightest change in pitch, loudness, or tone color will immediately produce a corresponding change in the wave line; but the extremest dynamic change, from perfect balance to acutest tension, will produce no effect whatsoever.

Actually the wave line is a picture not of a tone but of a vibration, the physical event that causes a tone sensation. There is a one-to-one correspondence between the acoustical properties of the tone on one hand and the physical event on the other. But there is no correspondence between the musical event and the physical event. The musical event seems to slip through the net of the physical world like light rays through a window pane.

If the origin of the dynamic qualities cannot be physical it must be psychological—it must be habit. This is the conclusion usually drawn. We have heard certain typical tone sequences so often that by now, when we hear music, we necessarily associate certain expectations with the tones; we feel satisfaction or dissatisfaction according as the expectations are

or are not fulfilled; these feelings we project into the tones and call them dynamic qualities. A rash conclusion this is indeed. If there is no music without dynamic qualities, the dynamic qualities cannot arise from our getting habituated to music. The fact that we develop habits when listening to music does not prove that music originates from habits. There is no evidence to support this thesis—except habit again, an old habit of thought, namely, to assume that if an event has no physical cause it must have a psychological cause; in other words, if it is not objective it must be subjective.

Music is neither. The conventional distinction between physical and psychological, objective and subjective reality does not apply to it. How a thing can be neither the one nor the other and yet still exist—in other words, how music is possible—remains a major problem for philosophy to clarify.[1]

The Octave

The beginning of another hymn will acquaint us with some more tones:

Seven different tones supply the material for this tune. Lined up according to pitch, they are . Our ear tells us that the tone towards which the others are directed and which appears perfectly balanced, the tone $\hat{1}$, is do . Do, re, mi, fa, sol correspond to $\hat{1}$, $\hat{2}$, $\hat{3}$, $\hat{4}$, $\hat{5}$ of the former example: la and ti are new.

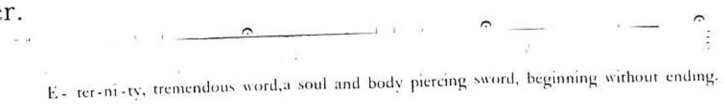
The first thing we notice is the difference in the tone $\hat{3}$, compared to the corresponding tone of the former tune. If we listen to $\hat{1}$–$\hat{2}$–$\hat{3}$ as it sounds here, do-re-mi and as it sounded there, la-ti-do the difference stands out most clearly. The characteriza-

[1] This problem is the topic of a separate study by the author (*Sound and Symbol*, Bollingen Series XLIV. New York: Pantheon Books, 1956).

tion of the dynamic quality of $\hat{3}$ given before would fit either case; the difference is of another order. It is what is called a difference of *mode;* the mode of the first tune is *minor,* of the second, *major.* We merely mention the phenomenon here and postpone its discussion for a later chapter.

The two new tones prove to the ear that they belong dynamically to the unbalanced type. La, which we identify as $\hat{6}$, seems to lean towards $\hat{5}$; this tendency stands out particularly when we hear the sequence $\hat{1}$–$\hat{6}$, which sounds somehow like an overshooting of the mark $\hat{5}$. (There is a certain ambiguity about the dynamic meaning of $\hat{6}$ which will be explained shortly.) The other tone, ti, or $\hat{7}$, points most sharply to the neighboring $\hat{1}$, voices the most urgent desire to move on to this tone.

What happens if we look for more new tones above $\hat{6}$ or below $\hat{7}$? The beginning of the following hymn tune has the answer.



The tune begins on $\hat{1}$, ascends through the intervening tones to $\hat{5}$, takes a new start on $\hat{5}$ repeated, continuing the ascending motion through $\hat{6}$ and beyond. The first tone after $\hat{6}$ is not a new tone; we recognize its dynamic quality: it is the same as $\hat{7}$. We hear its desire to move on to the next higher tone, and the tune complies with this desire. The tone that follows turns out to be $\hat{1}$ again, another $\hat{1}$, the same tone on a higher pitch level. ($\hat{1}$ above $\hat{7}$, as in the move ti-do, is often designated $\hat{8}$, and the move, $\hat{7}$–$\hat{8}$—with the understanding that $\hat{1}$ and $\hat{8}$ are two different symbols for the same dynamic quality.) If the ascending motion were continued further, beyond this higher $\hat{1}$, another $\hat{2}$ would appear, then another $\hat{3}$, and so on. If we look in the other direction, below ti, $\hat{7}$, we obviously get another $\hat{6}$, then $\hat{5}$, and so on.

This shows that after a certain pitch distance has been traversed, upward or downward, no more new tones appear; the old ones keep returning. The pitch distance at which this happens is called the *octave.* Thus, the tonal material of our music must be contained within the limits of an octave.

We have reached this limit. Seven tones, dynamically interrelated: one of them functioning, audibly functioning, as the center of action; the others directed, audibly directed, towards that center, each one in its own, almost personal way; some immediately, as $\hat{2}$ to $\hat{1}$, or $\hat{3}$ to $\hat{1}$, others via a mediating tone, as $\hat{4}$ to $\hat{3}$ and through $\hat{3}$ to $\hat{1}$, $\hat{6}$ to $\hat{5}$ and through $\hat{5}$ to $\hat{1}$. This is the stuff all our music is made of—at least all our music up to the end of the last century, when the transition from seven tone music to twelve tone music began.

The Scale

The series of seven tones, beginning with $\hat{1}$, plus the return of $\hat{1}$ as $\hat{8}$, is called a *scale.* The seven tone scale represents the tone selection, the tonal system of Western music.

The scale is not the beginning of music. Figuring out a scale first, then building melodies from the material of the scale: this is not the way music develops. People sing. Melodies appear. Scales are later theoretical abstractions, the result of reflection about the tonal material used in melodies.

We take a scale, e.g. $\hat{1}\ \hat{2}\ \hat{3}\ \hat{4}\ \hat{5}\ \hat{6}\ \hat{7}\ \hat{8}$ do-re-mi, etc., and consider the dynamic aspect of this tone sequence.

The dynamic situation between the first two tones, represented graphically, is this: do-re Accordingly the first move in the scale, the move $\hat{1}$–$\hat{2}$, is a move *against* the acting force, a move *away from* the center of action.

The dynamic situation between the last two tones of the series is this: ti-do. Accordingly the last move in the scale, the move

$\hat{7}$–$\hat{8}$, is a move *with* the acting force, a move *towards* the center of action. In the course of the movement along the scale the relation of the movement to the acting force has been reversed. Thus, the relation of the movement to the acting force at the end has turned into the opposite of what it was at the beginning. Where is the turning point?

We know that all tones between $\hat{1}$ and $\hat{5}$ look back towards $\hat{1}$. Up to $\hat{5}$, the relation of the movement to the acting force remains unchanged; it proceeds *against* the direction of the force, *away from* the center. $\hat{5}$ itself, however, seems to look not only back to $\hat{1}$ but also ahead to $\hat{8}$: the move $\hat{5}$–$\hat{8}$ satisfies the will of the tone $\hat{5}$ just as well as the move $\hat{5}$–$\hat{1}$. This indicates that beyond $\hat{5}$ we move *towards* $\hat{8}$, the center, *with* the acting force. $\hat{5}$ is the turning point.

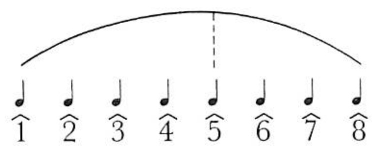
This explains the ambiguity of the tone $\hat{6}$ mentioned before. In one sense, $\hat{6}$ appears to lean towards $\hat{5}$, the next better balanced tone; in another sense, it is an intermediate station in the larger movement from $\hat{5}$ to $\hat{8}$, directing us onward through $\hat{7}$ to the destination.

The exceptional position of $\hat{5}$ among the tones dependent on $\hat{1}$ becomes now apparent. It is a sort of counter-pole to $\hat{1}$: it marks the greatest possible distance, dynamically, from the center. If we go beyond $\hat{5}$ we approach again the center—on the other side of $\hat{5}$. All tonal action can be said to take place between the extremes $\hat{1}$ and $\hat{5}$; the relationship between these two tones establishes the over-all framework for our tonal language.

The spiral line, ascending continually and returning after every turn to points exactly above those passed before, has often been suggested as a graphic symbolization of the octave structure of the pitch series. As a representation of the acoustical phenomena, this is satisfactory; as a representation of the dynamic, that is, the musical phenomena, it is not. It fails to express the sense of arriving at a destination that we experience

when the movement along the scale reaches $\hat{8}$. Something clicks here; but no sign of any clicking shows in the even course of the spiral.

The following diagram of a curve seems better suited to express graphically the movement through the seven tone scale—considered dynamically—as a musical phenomenon:



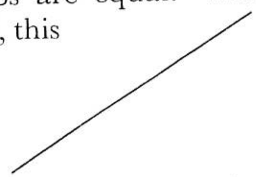
The strange fact of the octave—some authors have rightly spoken of "the miracle of the octave"—is responsible for the paradoxical situation that the movement along the scale brings into the open. It is a unique situation, one which has no parallel anywhere. Where else would we, by moving on continuously in a given direction, *return* to our starting point?—by increasing the distance from the starting point, *approach* that point again? For dynamically, and that is musically, $\hat{8}$ *is* $\hat{1}$, there is no mistaking the testimony of our ear regarding the sameness of start and goal of this movement. Going away means coming back; advancing is returning; movement ahead leads to the origin of the movement: these are statements of concrete fact in music.

It is true that there exist spatial parallels to this situation. If I move in a circle I shall always return to my starting point. If I walk out of the front door of my house and continue walking straight ahead in the same direction I shall in due course enter my house by the back door. But there is no need to move in space that way; we can do it, we are not constrained to do it. In music, however, there is no escaping the octave structure of the tonal world. Here every movement which is not prematurely interrupted must with inevitable necessity return to its starting point; either it turns back and so returns of its own will, as it were, or it goes on in the same direction and runs into the octave. There is no way of escaping the dynamic center by moving away from it in pitch. After a while it will have caught up with us, and we shall discover that what we thought we had left behind is already there, ahead, waiting for us.

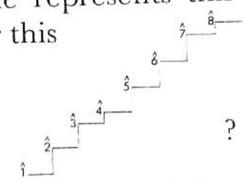
There is perhaps no other single factor that more deeply influences the ways of our music than the octave and all it stands for.

Whole and Half Tones

Do we hear the scale sequence do-re-mi, etc. as an even or uneven movement—"even" in the sense that the pitch distances of the successive steps are equal? Which line represents this movement more truly, this



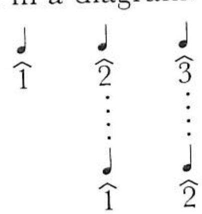
or this



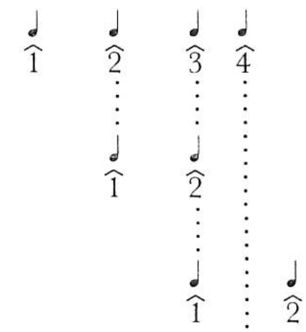
?

Most people whose reaction is not influenced by knowledge will have the impression that the scale movement proceeds evenly, by equal steps. They will reject the jagged line as a proper symbol. How could we test the correctness of this impression?

We know that a given pitch can assume any dynamic quality. Let us take the second tone of our scale, re, and imagine it as having the quality $\hat{1}$. Using a piano keyboard, we look for the tone which, if re were taken as the first tone of a new scale, would appear in the position of $\hat{2}$. We find this to be mi, which is the same tone as the tone $\hat{3}$ of our original scale. This shows that the two tones $\hat{2}$ and $\hat{3}$ can equally function as $\hat{1}$ and $\hat{2}$; in other words, that the pitch distances $\hat{1}$–$\hat{2}$ and $\hat{2}$–$\hat{3}$ are the same. Expressed in a diagram:

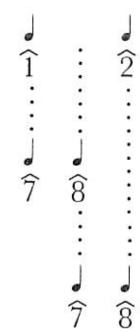


We go on and imagine the third tone of the original scale, as $\hat{1}$. We try whether the original $\hat{4}$ can serve as $\hat{2}$ to the new $\hat{1}$. It cannot. The tone is too low. The pitch distance $\hat{3}$–$\hat{4}$ is not the same as $\hat{1}$–$\hat{2}$ or $\hat{2}$–$\hat{3}$, it is smaller. The tone $\hat{2}$ we are looking for now lies beyond the original $\hat{4}$. In the symbols of the diagram:



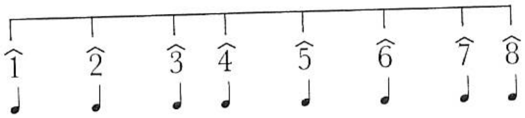
If we continue in this manner we shall find that $\hat{4}$–$\hat{5}$, $\hat{5}$–$\hat{6}$, and $\hat{6}$–$\hat{7}$ are all the same as $\hat{1}$–$\hat{2}$ or $\hat{2}$–$\hat{3}$, but that $\hat{7}$–$\hat{8}$ is again smaller, the same as $\hat{3}$–$\hat{4}$. We realize that the steps of the scale, in terms of pitch distance, are of two different sizes.

By how much is the larger size larger than the smaller? If we attribute to the tone $\hat{1}$ of the original scale the dynamic quality $\hat{7}$, the corresponding $\hat{8}$ will lie between the original $\hat{1}$ and $\hat{2}$. If we repeat this, and give to that $\hat{8}$ the quality $\hat{7}$, the new $\hat{8}$ which corresponds to the new $\hat{7}$ will be the same tone as the original $\hat{2}$.



Two $\hat{7}$–$\hat{8}$ steps in succession add up to one $\hat{1}$–$\hat{2}$ step. The smaller pitch distance in the scale seems to divide the larger in two equal parts.

The larger pitch distance is called *whole tone,* the smaller *half tone*. This gives us the following formula for the pitch pattern of our scale: two wholes, one half, three wholes, one half; or, translated into a spatial pattern:



The word "tone" in the terms *whole tone* and *half tone* does not stand for what it usually means, namely, an auditory sensation of a definite pitch. Here it refers not to one pitch but to a distance between pitches. This is in line with the original meaning of the Greek word "tonos," which is translated "tension." If we apply this meaning to certain commonplace statements and instead of "tones" read "tensions"—"tensions" are the material of music; music speaks a language of "tensions"—the sentences seem to express more of the true nature of music. It has indeed been said that our hearing of melody is not a hearing *of* tones but *between* tones—that music occurs not in the tones but between them.

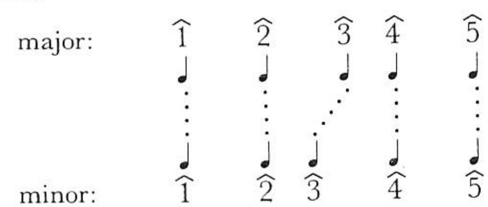
The Diatonic Order. The Modes

The pattern of our scale as shown in the last diagram—two whole tones, one half tone, three whole tones, one half tone—does not strike the mind's eye as distinguished in any easily discernible way. Still, this selection and distribution of seven tones has the unique distinction of representing the tonal order from which the music of Western civilization has grown, the *diatonic order.*

There is only one diatonic order, but there are different diatonic scales, each one having its peculiar pattern and producing its peculiar tonal expression. The differences are called differences of *mode.*

We have mentioned in passing one such difference, that of major and minor. The scale that we have analyzed is the diatonic scale, major mode, or more briefly, the major scale. The pattern we have extracted from it is the pattern of the major scale. What is the pattern of the minor scale?

When we compared the tones $\hat{1}$–$\hat{2}$–$\hat{3}$–$\hat{4}$–$\hat{5}$ as they appeared in the tunes of our first two examples (la-ti-do-re-mi in the first, which we called minor, do-re-mi-fa-sol in the second, which we called major), we could pin the difference to the change in character of the tone $\hat{3}$. No change was observed in the other tones. We see now that the tone $\hat{3}$ of the minor series, , is lower, and closer in pitch to $\hat{2}$, than the tone $\hat{3}$ of the major series, . From the pitch pattern of the major series, which we know, we can infer that of the minor series. The following diagram shows the two patterns:



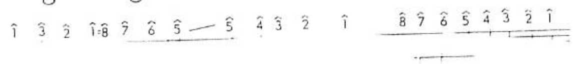
We see that the whole difference between major and minor hinges on a small change of pitch of the one tone $\hat{3}$. A whole tone between $\hat{2}$ and $\hat{3}$ makes the mode major, a half tone, minor. Consequently $\hat{3}$–$\hat{4}$, the half tone in major, must become a whole tone in minor. Strangely enough the shift in pitch does not affect the dynamic quality of the tone $\hat{3}$; in minor as in major it shows the characteristic half-balanced state with a slight but marked inner tension pointing towards $\hat{1}$.

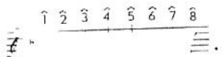
What about the rest of the minor scale?

[See "There is a Reaper"]

This is the tune of an old German folk song, "Reaper Death." At this point we consider only its first and last phrases. Taken together, they give us the complete seven tone series of the minor scale in descending direction. For clarity's sake we transpose the beginning one octave up:

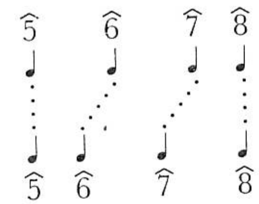
See Fig. 1



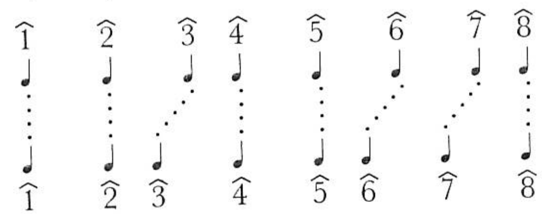
In ascending direction: 

We know the pattern from $\hat{1}$ to $\hat{5}$. What about $\hat{6}$ and $\hat{7}$?

The difference of the sequence $\hat{5}$–$\hat{6}$–$\hat{7}$–$\hat{8}$ here, mi-fa-sol-la and in major, sol-la-ti-do, is striking to the ear. Both $\hat{6}$ and $\hat{7}$ are lower in pitch; the half tone between $\hat{7}$ and $\hat{8}$ in major has disappeared and is replaced by a whole tone; and the pushing down of $\hat{6}$ produces a half tone between $\hat{5}$ and $\hat{6}$ where there was a whole tone in major. These are the two patterns:

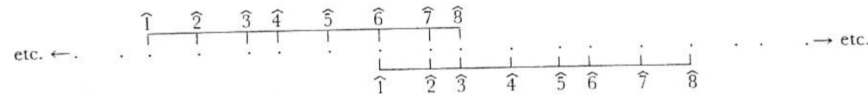


The disturbance of the dynamic picture created by this change of pitch will be discussed later. Here we mention only that in minor as in major $\hat{6}$ and $\hat{7}$ are both unbalanced tones. Here, finally, are the complete patterns of both scales:

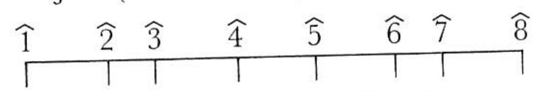
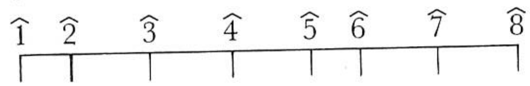
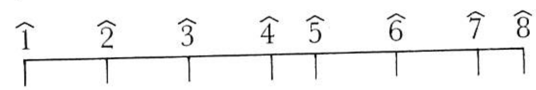


Two diatonic scales, each having its own distinctive pattern. How can we reconcile this with the statement that there is only one diatonic order of tones?

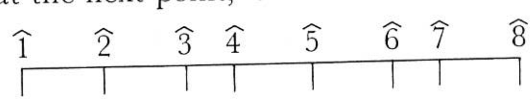
We picture the diatonic pattern—two wholes, one half, three wholes, one half—extending from octave to octave through the tonal expanse:



We see immediately that both scales are part of this picture. They are merely two different cuts, so to speak, from the basic pattern—different as a consequence of their starting (that is, fixing their tone $\hat{1}$) at different points of the order. The minor scale has its tone $\hat{1}$ at the point of the tone $\hat{6}$ of the major scale; the major has its $\hat{1}$ where the $\hat{3}$ of the minor is situated. It is not the major pattern which contains the minor pattern, or vice versa; it is the diatonic pattern which contains both in its framework.

It contains still other scales. The scale could begin at any point. Starting, for instance, at the point corresponding to $\hat{2}$ of major ($\hat{4}$ of minor), we get the following scale pattern:  ; if we start at the next point, corresponding to $\hat{3}$ or $\hat{5}$ respectively, the result is this:  ; if $\hat{1}$ is set where major has its $\hat{4}$, we get this scale: 

beginning at the next point, it is this:



These four scale patterns are not mere theoretical possibilities. They represent four distinctive modes of tonal expression, the Dorian, the Phrygian, the Lydian, and the Mixolydian—modes that dominated the music of the Middle Ages and that went out of existence with the advent of the major and minor modes (originally called the Ionian and the Aeolian) during the early seventeenth century. To a certain extent they have for some time now been in the ascendancy again, while the turn has come for major and minor to go out of existence.

As the main body of our musical heritage belongs to the major-minor period, we restrict our study to these two scales as representatives of the diatonic order.

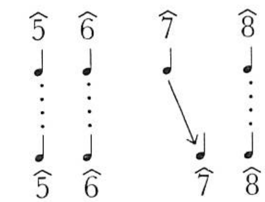
Minor

Taking the major scale as our example, we have read the meaning of its movement as: going away from the center, drawing closer to it again, and finally reaching it (in the guise of its octave replica) as the goal, with a sense of finality.

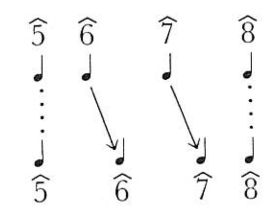
In the minor scale, la-ti-do-re-mi-fa-sol, we do not hear any of this. To be sure, we hear the going away from the center between $\hat{1}$ and $\hat{5}$; but as the movement proceeds beyond $\hat{5}$ there is no feeling of drawing closer to a destination, of approaching a goal, and particularly no sense of having reached a goal, no sense of finality as we arrive at $\hat{8}$. Why?

If we hear $\hat{8}$–$\hat{7}$ in major and stop at $\hat{7}$, the tone pulls back quite sharply to $\hat{8}$, audibly voices its desire to return there. If we listen to $\hat{8}$–$\hat{7}$ in minor, we hear nothing of that sort. The tone $\hat{7}$ seems here to have been pushed away from $\hat{8}$ so strongly that this impulse outweighs the attraction of $\hat{8}$. $\hat{7}$ in minor does not point towards $\hat{8}$ but rather away from it. As to $\hat{6}$, it sounds as if it were leaning in one direction only, towards $\hat{5}$; there is no indication of an ambiguity like that of $\hat{6}$ in major, no feeling whatever of being a station on the road, the downward incline, from $\hat{5}$ to $\hat{8}$. Since nothing in these two tones, $\hat{6}$ and $\hat{7}$, points to $\hat{8}$, it is clear that the sequence $\hat{5}$–$\hat{6}$–$\hat{7}$–$\hat{8}$ cannot convey the meaning of drawing closer to the destination $\hat{8}$, cannot stress $\hat{8}$ as the goal of the motion.

In order to impart to the movement through the minor scale the same kind of conclusiveness it has in major, something must be done to the stretch between $\hat{5}$ and $\hat{8}$. The main obstacle seems to be the lowered pitch of $\hat{7}$, its greater distance from $\hat{8}$. If $\hat{7}$ is a whole tone distant from $\hat{8}$, the step $\hat{7}$–$\hat{8}$ cannot have the affirmative character, the conclusiveness and finality characteristic of the satisfaction of a clear and strong desire, and the power to stress $\hat{8}$ as the point of destination—all of which it has in major. Therefore, to produce these effects in minor, too, it is necessary to raise the pitch of $\hat{7}$ to the half tone distance from $\hat{8}$:

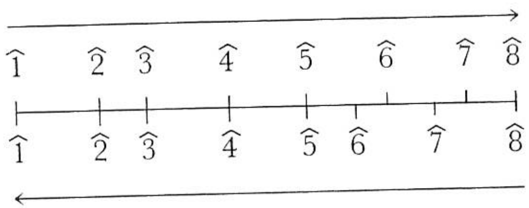


As we see—and hear—this creates an oversized pitch distance and an awkward step from $\hat{6}$ to $\hat{7}$. To equalize the unevenness, $\hat{6}$ must follow suit and pull up closer to the new $\hat{7}$:



With this distribution of tones the movement from $\hat{5}$ to $\hat{8}$ will have the same effect, the same meaning it has in major; in fact, it *is* the same movement, the pitch pattern is the same. To state it briefly, then: If the minor scale in ascending direction is expected to express the same sense of conclusiveness manifested by the major scale, it must borrow the tones $\hat{6}$ and $\hat{7}$ from major. (In descending direction there is no problem as the last move, $\hat{2}$–$\hat{1}$, is the same in both scales.)

This gives us two alternate positions of the tones $\hat{6}$ and $\hat{7}$ in minor, and two alternate scale patterns: (1) the normal, or diatonic, usually associated with the descending direction, and (2) another, with elements from major mixed in, which usually appears in connection with the ascending direction. Here they are:



To sum up: The difference of major and minor appears at three points of the scale pattern: $\hat{3}$, $\hat{6}$, and $\hat{7}$. Yet while the position of $\hat{6}$ and $\hat{7}$ can shift without producing a change of mode—minor can borrow them from major and still remain minor—a shift in the position of $\hat{3}$ will immediately produce the change. This is the decisive tone; the difference of major and minor is concentrated in it. The moment one hears $\hat{3}$ one knows which of these two modes prevails.

Declaring the Center

We have tentatively defined melody as a sequence of dynamically interrelated tones. This does not mean that, first, dynamic qualities are given as a sort of material, and second, melodies are built from this material. The relation of melody to dynamic quality is of a more intricate type. The tones themselves by their movement call into action the forces which then shape the movement, create context, meaning, structure, unity: a melody.

Dynamic qualities are audible relations of tones to other tones and ultimately to one central tone. Without a dynamic center there would be no dynamic qualities. So first there must be a center. How does it come into being? How do we know a certain tone is $\hat{1}$? How does a tone tell us that it is the center?

Obviously not just by sounding. If we hear a single tone we

may feel inclined to take it for $\hat{1}$; but as long as nothing else happens this is mere presumption. The fact that the sun is the center of our planetary system is not established at the place of the sun itself but way out in space where the planets through their movements confirm the sun's ruling power. Similarly the central position of a tone will be established not by that tone itself, its mere presence, but by the events occurring around it.

The way the tones move—the steps they choose, the paths they follow up and down, the turns they take—spells out to the ear which tone is $\hat{1}$. To every dynamic center there belongs a dynamic field of which it is the center. Music is movement of tones in dynamic fields. From the movement of the tones the ear gathers the configuration of the field; we hear in what direction the tones are oriented, where they point. By the other tones' pointing at it, the tone $\hat{1}$ is revealed to the ear as the center of action.

There are many ways for a melody to state the dynamic center, some obvious and conventional, others more remote and intricate. Most complete and unequivocal is the one chosen by the "Eternity" hymn: (first phrase) . Here we have all the seven tones of the system, arrayed in scale pattern, beginning and ending with $\hat{1}$: we cannot want more. The "Allelujah" hymn gets along with a briefer statement (first phrase), $\hat{1}$–$\hat{2}$–$\hat{3}$–$\hat{2}$–$\hat{1}$. It has to be repeated, though, for stronger confirmation. The hymn tune (hymn #2) does not begin with $\hat{1}$, but chooses to declare its center by a $\hat{3}$–$\hat{2}$–$\hat{1}$–$\hat{2}$–$\hat{3}$–$\hat{4}$–$\hat{5}$ motion. Brief formulas—like half tone down and up again, which the ear interprets as $\hat{8}$–$\hat{7}$–$\hat{8}$, or whole tone up and back, $\hat{1}$–$\hat{2}$–$\hat{1}$—may be used; but they are not unambiguous. The half tone move could well be $\hat{3}$–$\hat{2}$–$\hat{3}$ in minor, the whole tone move $\hat{5}$–$\hat{6}$–$\hat{5}$ in major, or what not. However, this very ambiguity may occasionally be exploited for the sake of a special effect: the ear is allowed to incline towards a certain dynamic interpretation that a subsequent event proves wrong, thereby retroactively changing the meaning of the preceding moves. If this happens on a larger scale, the effect can be quite shattering (the beginning of Beethoven's Ninth Symphony is a famous example). Ordinarily a piece will declare its center right away; but sometimes a definite center will emerge only after a long search that keeps the hearer in a prolonged suspense. The first movement of Schumann's Piano Fantasia finds its center only at the very end.

The Making of Melody

A close study of the tune of the "Allelujah" hymn will demonstrate concretely the making of a melody through the action of the tonal forces.

The first move, la-ti-do-ti-la ascending and descending, away-from-and-back-to- la , makes that tone appear as $\hat{1}$. The repeating of the move confirms la as center. What next?

mi , $\hat{5}$: the counter-pole. We hear the direction of the tone towards $\hat{1}$, the tendency to return there.

The melody complies with this will: mi-re-do-ti. Not all the way, though. Just at the point where the tension is most acute, where the will to make one more move is most outspoken—at $\hat{2}$, —the movement comes to a halt. Accumulated suspense. What will happen now?

What happens is the very opposite of what the last tone wanted to happen. The movement turns around, draws away from $\hat{1}$, goes up, through $\hat{3}$ and $\hat{4}$, to reach $\hat{5}$ again:

The going is slower now than on the downward stretch, the movement seems to take a new breath before each step, ti-do, do-re, re-mi, we almost feel how it works against the pressure of the acting force.

With the reaching of $\hat{5}$ the opposition to the acting force is spent, the movement once more conforms to the will of the tone

$\hat{5}$, and the second try succeeds: mi-re-do-ti-la . The way is completed, the goal reached:

See Section A

Through the whole second half of this melody, from the moment the movement came to a stop at $\hat{2}$, we are waiting for the fulfillment of the expectation aroused by that tone. Its tension underlies and ties together everything that follows and is released only at the end, with the entry of $\hat{1}$.

The second part of the tune is on the whole a repetition of the first, but it does not begin as the first did. There is no need now for the repetition of a short phrase whose main function it was to establish $\hat{1}$. Still, the beginning should again be the repetition of a short phrase. So we begin with the second phrase, mi-re-do-ti, and repeat this.

This means that now we have *two* unsuccessful tries; twice the movement is stopped at $\hat{2}$. As a consequence, the tension of this tone is greatly enhanced. What follows is therefore more than a mere repetition of the second half of the first part: the ascending phrase works against a stronger force, carries greater weight. Finally, the entry of $\hat{1}$ brings a more emphatic release, makes a stronger punctuation than at the midway mark—which is as it should be:

See Section B

We see two things here. First, the action of the tonal forces organizes the tune, holds it together, gives it its meaning. Second, the tones in their movements are not simply subjected to the acting forces, as inanimate bodies are subjected to the force of gravity; they are free to move with or against the acting forces, as the animate body of the dancer is free to move with or against the force of gravity.

The tonal forces do not determine the tonal movement, but they do determine another thing, and this in the strictest sense: the *musical meaning* of that movement. In our tune the movement was free any time to proceed from $\hat{2}$ to $\hat{1}$ or to any other tone it chose; but once it did take the step $\hat{2}$–$\hat{1}$, nothing on earth could give that move another meaning than "in conformity with the acting force, reaching the center," and make us hear in it anything but a move from an unbalanced to a perfectly balanced state. Just as strictly and unequivocally will the meaning of any other move be determined by the prevailing dynamic situation.

(Mendelssohn once said that the tonal language is too precise to be translated into words. We see the truth of this statement. An author has a certain margin of freedom to change the meaning of his material, words; a composer has none.)

The forces that we have observed in action, which are responsible for the making of this simple tune (a very beautiful, an excellent tune in all its simplicity), are the same ones that form and unite the most complex musical organism. In this respect the difference between a folk song and a Beethoven symphony is only one of degree. The increase in complexity may be comparable to that from grade school arithmetic to calculus; the principle is the same.

Background and Foreground

In the "Allelujah" tune every single tone represented, as it were, a station on the main road of the melodic movement. For instance, when the melody moved from $\hat{5}$ through $\hat{4}$ and $\hat{3}$ to $\hat{2}$, it did so in the most simple, straightforward way, using one tone for every station,

This is by no means the only way, nor is it the most usual way, for a melody to move. We could call it the naive way, characteristic of music in the state of nature, of folk music. Rarely will it be met with in art music, which develops more complex types of movement. Occasionally a folk tune, too, will show a higher degree of complexity. The "Reaper Death" melody is an example.

Its first phrase, , takes us after a little tour indicating the center, $\hat{1}$–$\hat{3}$–$\hat{2}$–$\hat{1}$, down through $\hat{7}$ and $\hat{6}$ to $\hat{5}$ and stops there; the counter-pole is stated. This is done in the simple manner with which we are familiar.

The second phrase begins by taking up $\hat{5}$ to the higher octave: , the tone is thereby put into strong relief, and rightly so since all that follows hinges on it. Next, the tones are released on their downward journey, towards $\hat{1}$; they get no farther than $\hat{3}$: . The sum of this is a movement from $\hat{5}$ to $\hat{3}$. Not a simple $\hat{5}$–$\hat{4}$–$\hat{3}$, though. If we spelled it out tone by tone it would read $\hat{5}$–$\hat{4}$–$\hat{3}$–$\hat{4}$–$\hat{3}$–$\hat{2}$–$\hat{3}$. This, however, is not a true representation of what we hear. For instance, nobody will hear in the first 'do' of the second phrase the dynamic quality $\hat{3}$. What we do hear is better represented this way: $\hat{5}$ → $\hat{4}$ → $\hat{3}$. It *is* a $\hat{5}$–$\hat{4}$–$\hat{3}$ movement; but the individual links of the movement are no longer single tones: they are small groups of tones. It is a movement not from tone to tone but from group to group, with each group comprising three tones and representing one station on the main road. (Obviously the rhythm contributes much to this understanding, but we do not consider the time factor here.)

Nothing conclusive has been achieved so far. The melody returns to $\hat{5}$ for another start. This time we get farther; but the movement is even slower and, one might say, more massive than before: the groups have grown to twice their former size:

When the movement arrives at $\hat{2}$ it seems not ready to take the final step. It skips $\hat{1}$ and drops down to the lower $\hat{5}$.

Once more, $\hat{5}$ is lifted to the higher level; and now only, returning to the simple manner of the beginning, the melody takes us all the way from $\hat{5}$ to $\hat{1}$.

With this we can give a reasonable account of the tonal story of this tune:

On the whole, it is a movement through the full octave, descending from $\hat{8}$ to $\hat{1}$. The first stretch, from $\hat{8}$ to $\hat{5}$, is traversed quickly and directly. The remaining part of the tune consists in repeated attempts to complete the journey. More roundabout ways are now chosen, first through groups of three notes, which gets us to $\hat{3}$, then through groups of six notes—this comes to a halt on $\hat{2}$. The last attempt again strips the movement to its bare essentials, one tone to one station, and succeeds.

We see from this example that a melody does not simply string tones together, one after another, as in a single file. In the middle section of the tune a kind of organic partition develops, the movement proceeds on two planes simultaneously. In the background, as it were, the succession of main stations; in the foreground the detours, the subordinate patterns by which station is linked to station.

The succession of main stations shall be called the *skeleton* of a melody. Its function in the life of the melody is exactly that of the skeleton in a living organism. Beauty, individuality, distinction do not reside in the skeleton but rather in that which the skeleton supports. Yet it is the skeleton that does all the supporting; without it, the whole thing would immediately collapse.

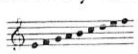
The normal listener will follow the thread of the "Reaper Death" tune without difficulty. It will make sense to him, connected sense. Since it is the skeleton that creates the connection, the ear must instinctively be aware of its existence and of the corresponding background-foreground structure of the melody. Since the beauty and the interest and in fact the very meaning of this tune, as of every tune, rests precisely in the relation between the flesh and blood of the musical foreground and the supporting skeleton of the background, the ear of any person who finds any interest or beauty or sense in this melody must somehow have understood that relation.

Listening to a melody we intuitively distinguish between the main road of the movement and the sideroads and detours that take us from one station on the main road to the next, elaborating, expanding, enriching the movement. The ear is constantly engaged in forming smaller and larger sums of tones, according to the structure of the movement, grouping together what belongs together, keeping apart what should be kept apart. It does not simply register tone after tone, taking every tone at its face value; it evaluates every tone according to its function in the context of the melodic movement. At the start of line four, for example, the tone la is identical in pitch with $\hat{1}$; but it will not be heard as $\hat{1}$. The tone at this point is a link in the subordinate movement into which the tone $\hat{2}$ has expanded, a part of the group that as a whole represents the station $\hat{2}$ of the background movement; it does not function as $\hat{1}$ here. This the ear understands, and it evaluates the tone accordingly. If the ear were not capable of making these distinctions and judgments, the tune would not make sense.

What happens in this simple folk tune happens similarly, on an ever larger and more complex scale, in elaborate compositions. To the intellect, intent on laying bare the skeleton, on demonstrating the background-foreground structure, such compositions present increasingly difficult tasks. But it is not so with the ear, which will all along perform the proper operations with the same instinctive certainty, gathering tones into smaller and larger groups, groups and sequences into units of a higher order, separating and uniting in line with the organization of the tonal movement. This synthetic power of the human ear goes far beyond a mere reaction to a series of stimuli; it can be greatly developed by training, but in an elementary way it will function even without any training. If it were not for this power, there would be no music.

The Center Moves

We can look at a piano keyboard as a fair visualization of the diatonic order. The white keys represent the tones of that order. The alternating groups of two and three black keys, separated by one empty space, mark the places of the alternating two-whole-tone and three-whole-tone groups; the location of the half tones is indicated by the absence of a black key between two white keys.

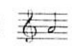
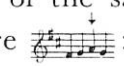
We identify the white keys by simple letter names—a, b, c, d, e, f, g—and represent them by the successive positions of the plain symbols on the staff, .

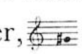
If the white keys correspond to the tones of the diatonic order, they must form a major scale. As we know the pattern of that scale we can locate its tone $\hat{1}$: it is the first of the three tones forming the two-whole-tone group (the 4th white key from the left on the sketch above). This tone is called c, the white keys give us the C major scale. The same keys must give us a minor scale, too. Its $\hat{1}$ will be located at the tone $\hat{6}$ of the major scale, the tone a; it is the A minor scale.

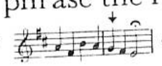
If our music is seven tone music, why do we need the addi-

tional tones, the black keys—five of them to every octave? The raising and lowering of the pitch of a piece, like the transposing of a melody to a higher or lower pitch level, could be taken care of by a mechanism, like that on a harp, that in one move changes the pitch level of the whole series. The following tune exemplifies the crucial event that necessitates the introduction of new tones without basically changing the nature of the seven tone system:

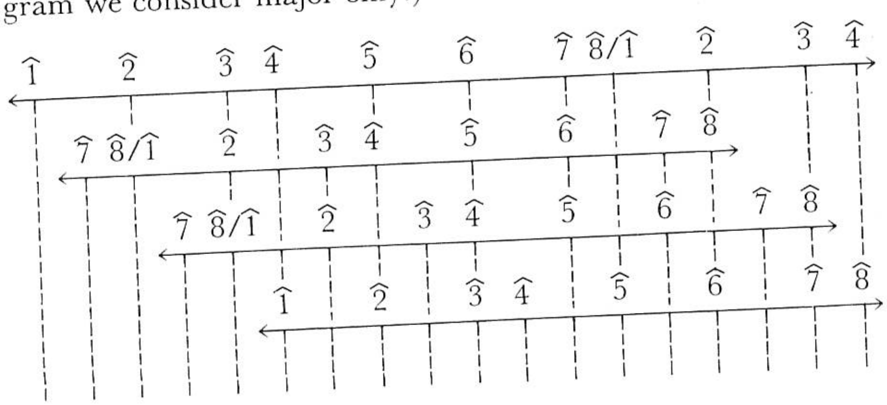
We have listened to the first part of the tune before. The center is . The movement proceeds without disturbance up to the point marked x. Here the ear feels a light shock. Something has happened.

We listen to the dynamic quality of the tone  in the measure following that event. The phrase comes to a completely satisfactory ending on it, the tone is perfectly balanced. We compare this to the dynamic quality of the same tone as it appeared shortly before, in the measure : we realize the difference. There, it was $\hat{5}$; now it is $\hat{1}$. This is the crucial happening: in the course of the movement of the melody the dynamic center has changed place, has moved from one tone to another.

How has this been brought about? Among the typical short ways by which the tonal movement may establish a tone as the dynamic center we have mentioned the move $\hat{7}$–$\hat{8}$, a half tone step upward. If the melody chooses this method to make $\hat{5}$ appear as $\hat{1}$, it must approach and reach this tone through an ascending half tone step. The lower scale neighbor of $\hat{5}$ in the original pattern, the tone $\hat{4}$, is a whole tone distant. Accordingly a new tone, half way between $\hat{4}$ and $\hat{5}$, must be introduced in order to achieve the desired result. This is the tone that caused the light shock to the hearer, . It breaks the prevailing seven tone pattern, or rather, it breaks the pattern loose from its mooring to the prevailing tone $\hat{1}$ and shifts it over to a new anchorage.

In a delicate way the movement itself has prepared us for that event. When it occurs we have not heard , the original tone $\hat{1}$, for almost 5 measures. In the measure immediately preceding it, , the failure of the movement to touch $\hat{1}$ was quite conspicuous. It seems as if the grip of the center on the movement were loosening; and immediately another tone takes advantage of the situation and attracts the movement towards itself, pushes itself in the position of $\hat{1}$. Shaken from its complacency by this event, the original center returns to active life and smoothly directs the movement back where it belongs, into *its* own orbit, by simply annulling in the following phrase the raised tone, returning it to its original pitch position: . The original order is duly restored.

As a tone is $\hat{1}$ not by itself but in relation to the other tones of the seven tone pattern, a shift in the position of $\hat{1}$ implies a shift of the whole pattern. The following diagram shows that such a shift will necessarily call forth new tones, that is, tones that were not parts of the original seven tone set. What the problem practically amounts to is the construction of new scales beginning at other tones than the original $\hat{1}$. (In the diagram we consider major only.)



First, the tone $\hat{2}$ of the original set should become $\hat{1}$. We know from an earlier observation that the tone $\hat{3}$ of the original scale can be used as second tone of the new scale. What about the new $\hat{3}$? It must lie a whole tone above the new $\hat{2}$, or the original $\hat{3}$; but the original $\hat{4}$ lies a half tone above $\hat{3}$. It cannot be used, it is too low to function as $\hat{3}$ in the new scale; a new tone, half way between the original $\hat{4}$ and $\hat{5}$ must be produced for the purpose. For the following tones the new scale can use pitches of the original scale; except for $\hat{7}$. Here again a new tone has to be created. Similarly, if the original tone $\hat{3}$ is made $\hat{1}$ of a new scale, it will be necessary to introduce new tones for the functions $\hat{3}$ and $\hat{7}$. If the original $\hat{4}$ becomes $\hat{1}$, a new tone is needed for $\hat{4}$ of the new scale.

There is no need to carry this further. The five whole tones of the diatonic order from which we started are now all filled up, the tonal expanse is evenly divided into equal pitch distances, the tones are evenly distributed at half tone intervals, twelve of them to the octave. Consequently the major scale pattern—or the minor scale pattern, or any diatonic pattern—can now be applied beginning from any tone; there are tones available for any position of the pattern.

The series of twelve tones is called the *chromatic scale*. When we hear it we notice a striking difference from the seven tone, the diatonic, scale. The dynamic qualities have disappeared. One tone is as good as another; no tone is audibly related to any other tone; all that remains is difference of pitch. The organization, the musical order has gone.

Looked at quantitatively, the chromatic scale with its even distribution of tones seems a much better order than the arbitrary diatonic arrangement. Musically, however, the even distribution spells the end of order. The chromatic scale is the musical equivalent of chaos.

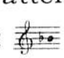
The expansion of the tonal material from seven to twelve tones does not replace the seven tone order with a twelve tone order. The twelve tones are no more than a kind of storehouse,[2] a reservoir where music finds the material needed to establish a diatonic order with any tone as center. Thus, the tonal movement wins an additional freedom, a momentous freedom: that of moving its own center at will. It is not only movement about a center but movement of a center. The latent tendency of every tone to establish itself as the center of dynamic action can make itself felt; the competition among tones for the central position in the dynamic field becomes one of the major topics of music.

Sharps and Flats

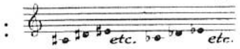
What should the new tones be called, and how should they be represented on the staff?

We take the C major scale as the starting point, as the zero series, so to speak: c, d, e, f, g, a, b, c. For every position of the center other than c—the mode remaining the same—one or more tones of this series will prove out of line with the requirements of the seven tone pattern.

Assume that the tone d has become $\hat{1}$. The tone e will then be $\hat{2}$; but f cannot serve as $\hat{3}$, since it is too low (e–f is a half tone, whereas $\hat{2}$–$\hat{3}$ in major is a whole tone). In order to get a tone that will function as $\hat{3}$ in the new context, the pitch of f must be raised—sharpened—by a half tone. The new tone we get that way is called f sharp. Symbol: .

Assume that f becomes $\hat{1}$. Tones g and a can be used as $\hat{2}$ and $\hat{3}$, respectively; but b cannot serve as $\hat{4}$, it is too high (a–b is a whole tone, $\hat{3}$–$\hat{4}$ in major a half tone). In order to get the tone that will function as $\hat{4}$ in the new context, the pitch of b must be lowered—flattened—by a half tone. The new tone is called b flat. Symbol: .

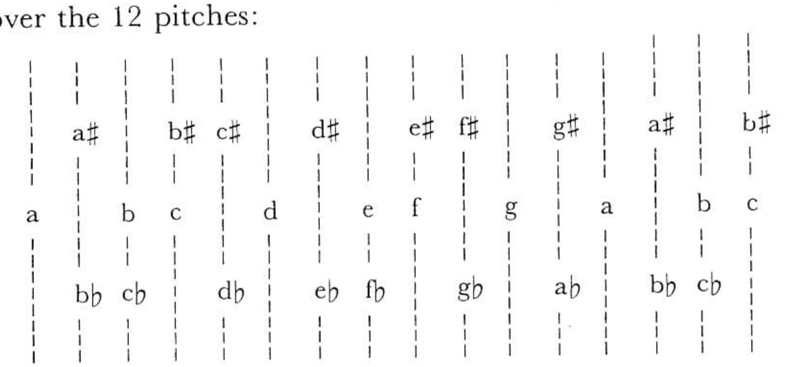
[2] In modern twelve-tone music, too, the chromatic scale represents an assembly of materials rather than an underlying order—materials needed for the building of tone rows.

We generalize: Whenever one of the tones of the zero series, c, d, e, etc., proves too low and must be raised in order to fit the requirements of the pattern in a new position, the tones thus introduced are called c sharp, d sharp, e sharp, etc.; whenever for the same reasons these tones must be lowered, they are called c flat, d flat, e flat, etc. Symbols: .

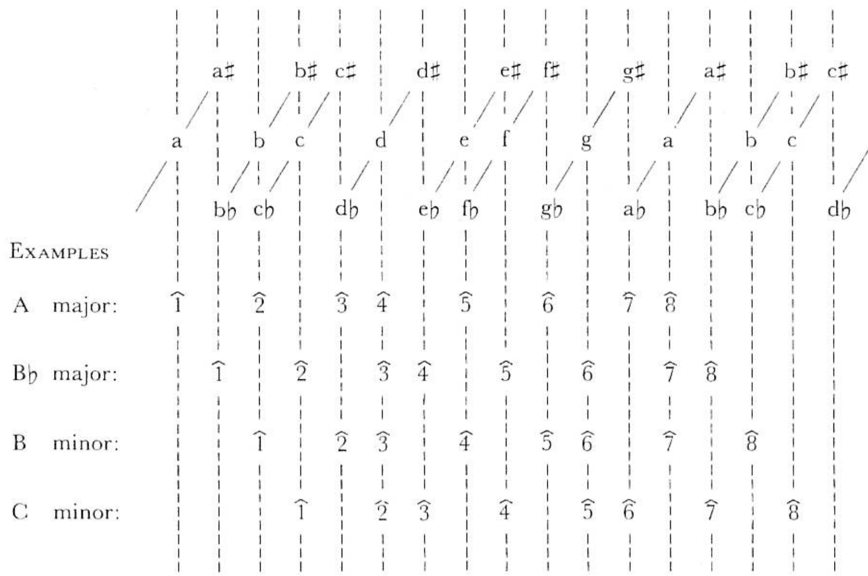
This seems to give us altogether 3 times 7, that is, 21 names, 21 symbols. Twelve pitches only are available. How are we to reconcile this?

Let us consider, for instance, the situation between c and d. If d is too high and must be lowered by a half tone, we get d flat. If c is too low and must be raised by a half tone, we get c sharp. Both c sharp and d flat are supposed to lie half way between c and d; they will coincide in pitch. One pitch can take care of both eventualities. If c is too high and must be lowered by a half tone, the tone b, which is a half tone below c, can take care of that. If e is too low and must be raised by a half tone, the tone f can supply the needed pitch. (A little cheating is involved in all this. The pitch distance we call a half tone is not exactly one half of a whole tone; c sharp is not exactly the same as d flat, c flat is not exactly b, and so on; but the difference is small enough to be neglected in practice. We shall hear more of this later.)

The following diagram shows the distribution of the 21 tones over the 12 pitches:



This looks disconcertingly confused. However, we have only to connect what belongs together, and the inherent order of the whole system will immediately emerge.



From this table we can read the names of the tones of any seven tone set—the set belonging to any given tone $\hat{1}$; in other words, we can read from it any scale, beginning with any tone, in any mode. The principle is this: Every letter station—the three "appearances" of the letter connected by diagonal lines—must be represented by one tone in the scale. No station shall be skipped, none used twice. Accordingly, for example, the A major scale spells as follows: a, b (not c flat, as this would skip the b station), c sharp (not d flat, because then the c station would not be represented), d, e, f sharp, g sharp, a. The B flat major scale: b flat, c, d, e flat (not d sharp, because then the d station would be used twice), f, g, a, b flat. The B flat minor scale (not shown in the diagram) would spell: b flat, c, d flat (one half tone above c, but not c sharp, because c has already been used), e flat, f, g flat (not f sharp, for the same reason), a flat, b flat. And so on.

Key and Signature

We open a volume of music and read on the title page: Symphony No. 4 in B flat major, opus 60, by Ludwig van Beethoven. What does this mean, "in B flat major"? The answer is: It indicates the *key* of this particular piece. What is a key?

Key, as a technical term in music, signifies the organization of the twelve tone material with reference to a particular tone as the dynamic center. A key is the selection of seven tones in accordance with the requirements of the pattern—major or minor mode, as the case may be—which puts one particular tone in the position of $\hat{1}$. Selecting seven tones out of twelve means at the same time rejecting five. Thus, every key draws a dividing line across the twelve tone reservoir, separating the seven tones that belong to it from those that do not belong. The seven chosen tones are called *diatonic,* the rejected, *chromatic* tones. The dividing line between diatonic and chromatic tones is not fixed but changes with the key; different tones are diatonic or chromatic for different keys.

(In one respect this statement is not quite accurate. A key rejects not five tones but five pitches. If we consider the 21 tones of our last diagram as the material from which a key selects its seven diatonic tones, there will be 14 rejects. However, some of them will practically coincide in pitch, while others will share the same pitch with diatonic tones; for instance, in the key of C major the tone c is a diatonic tone while b sharp, practically identical in pitch with c, would be a chromatic tone.)

The key is B flat major: this means that the tone b flat is the center, and the diatonic tones are the seven tones that make up the pattern, major mode, for b flat as $\hat{1}$; in other words, the tones of the B flat major scale. With one exception, to be mentioned later, no two keys have the same set of diatonic tones; each key makes its own selection. This selection, as we have seen in the scales, takes the form of replacing the tones of the zero series by their sharpened or flattened versions. The B flat major scale, for instance, came into being by replacing the tones b and e of the zero series by b flat and e flat. The A major scale changed the tones c, f, and g of the zero series into c sharp, f sharp, and g sharp.

The sharps or flats that belong to the diatonic tones of a key are called its *signature*. In our system of notation they are always put ahead of the text, at the beginning of every line. If a piece is in B flat major, every line will begin with . This says literally: As long as this signature stands here, every b symbol of the text means b flat, every e symbol e flat. In A major the signature is , the three sharps that we know the A major scale contains. (It will be explained later why they appear in this particular arrangement.) Actually the signature spells out all seven diatonic tones of a key: the tones of the zero series which are not flattened or sharpened will appear in the key in their unflattened-unsharpened, their natural version. In this sense the signature identifies the key.

Tonality

Musical thoughts may occur to a composer in a kind of abstract form, not fixed to particular pitches. As soon as he wants to express them in writing or playing, he will have to choose definite tones. He can think a melody in the major mode; he can write or play a melody only in a major key, that is, definitely localized in the tonal area, centered about a definite tone.

What determines the choice of key, apart from technical considerations such as the pitch range of instruments or the difficulty of execution, is a highly interesting and puzzling question for musical psychology and esthetics. A definite coordination seems to exist in many instances between the character of a piece and its key. Sometimes, as in certain typical arias of 18th

century opera, the choice is rooted in tradition. In other cases it appears as a distinctly personal factor. One can find a community of character among many D major pieces, of another character among many E major pieces, of Mozart, or Beethoven, or Bach; but Mozart's D major character is very different from Beethoven's, which is again different from Bach's. The problem is further obscured by the change of the pitch standard over the centuries, and in the case of Bach and his contemporaries and predecessors, by the different tunings of keyboard instruments (a piece to be played on a certain organ would have to be written in C major if one wanted it to sound in D major). However, on the whole and particularly from the middle of the 18th century on, one can assume that normally a musical idea would occur to a composer from the outset as located in a definite key, idea and key belonging intrinsically together.

The music that was to become his Fourth Symphony occurred to Beethoven as centered about the tone b flat, embodied in the key of B flat major. Are we to assume that no tones except the seven tones of the B flat major scale will appear throughout the piece?

We can expect the music of this symphony, as of any larger composition, to make ample use of the freedom of moving the center. This implies the introduction of new tones, tones that do not belong to the diatonic set of this key: chromatic tones.

In the tune quoted before (p. 45) as an example of shifting center, the tone , which brought about the shift, was a chromatic tone; it replaced the diatonic g, the tone $\hat{4}$ of D major, the key of the tune. Whenever the center changes, one or more tones of the original seven tone set of the key will be too high or too low and will have to be flattened or sharpened. New sharps and flats, in addition to those that belong to the signature, will then appear in the text. If a tone that is sharpened by the signature should prove too high and must be lowered, the sharp will be cancelled for the moment; for the opposite reason a flat of the signature will be momentarily cancelled. The symbol for cancellation of either sharp or flat is ♮, the *natural*. Occasionally a sharpened tone must be further sharpened, a flattened tone further flattened; this brings in double sharps, ×, and double flats ♭♭. Together these symbols—sharps, flats, naturals, double sharps, double flats—are called accidentals. Between the sharps and flats that belong to diatonic tones and make up the signature, and the accidentals that mark chromatic tones, there is this distinction: the former are set apart, at the beginning of the line, and are valid as long as the key is in force, while the latter are written immediately before the symbol to which they refer and are valid for one measure only.

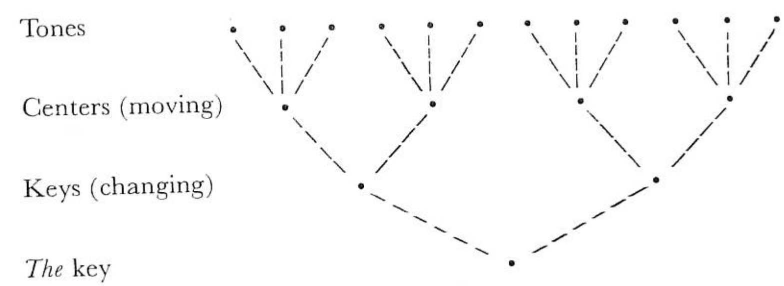
If in the course of a piece of music the center can move, what sense does it make to talk of key, which means that one definite tone is the center?

The notion of key does not imply that a certain tone will be $\hat{1}$ all through the piece. What it does imply is that the movements of the center will not be arbitrary, that in some way they will manifest the presence of a power behind the stage, as it were, a center of centers: this is the tone $\hat{1}$ of the key. Whatever power any other tone may hold in the course of the piece is only temporary, delegated power that in due time will return to the true sovereign. In order to understand how this happens one has to observe in detail the movements of the center throughout a piece: a task for a later chapter.

This is not all. Key itself is not the ultimate of stability either. In the course of a larger composition the key may change, too. However, the changes of key will only repeat on a larger scale the story of the movements of the center. There will be nothing arbitrary about them; from the ways of the changing keys a definite pattern will emerge and clearly point at one of them as

the key of keys, the ultimate ruler. This, then, is *the* key of the piece, the one named in the title.

The following is a graphic summary of these relations:



(In actual music these levels are not strictly separated. Some changes of center are so swift that they can hardly be counted as such; sometimes a new center will entrench itself so strongly that it practically amounts to a change of key.)

All this together, the relation of tones to centers, of centers to keys, of keys to a super-key, is implied in the term *tonality*. In a superficial sense tonal music simply means music built on audible relations of tones to a central tone, on the dynamic qualities as we have observed them. In a truer sense tonality means what the last diagram shows: the manifestation of the power of one tone through all the levels of a complex musical organism. Tonal music in this sense culminates in the works of the great masters of the 18th and 19th centuries. Towards the end of the 19th century the disintegration of tonality begins. The first link to be cut is that between the changing keys and the super-key: music is written in keys but not in a key. The foundation is no longer stability; it is change. Later, key dissolves, the moving centers cut themselves loose from the common point of reference. Finally, the ever more rapid succession of changes of center put the notion of center itself in doubt; the diatonic order disappears, and with it all tonal relations as we have known them. This is *atonal* music, the product of our century; in the perspective of tonality the purest negation; in its own perspective a search for new kinds of tone relations.

Key Relations

Different keys do not meet each other, so to speak, as absolute strangers. Delicately shaded degrees of kinship exist between them and make themselves felt to the ear. Stated in the most general terms, the relationship of two keys is determined by the things they have in common.

We have mentioned that there is one exception to the rule that every key has its own set of seven diatonic tones. We know that the diatonic pattern furnishes the framework for various scale patterns, each beginning at a different point of the diatonic order, each corresponding to a particular mode. Thus a given seven tone set provides the tonal material for more than one scale—specifically, for one major and one minor scale—as we saw the zero series provide the material for the C major and A minor scales, with the tone $\hat{6}$ of major supplying the pitch for $\hat{1}$ of minor, and $\hat{3}$ of minor coinciding in pitch with $\hat{1}$ of major. In the same way every seven tone set can give birth to one major and one minor scale, beginning with different tones; this means that one seven tone set can provide the tonal basis for two different keys, one major, one minor. This particular pair of keys of different mode that have one seven tone set in common is called the *relative major and minor* or, briefly, the *relatives*. Since the tone selection of a key is determined by the signature, relative major and minor will always have the same signature. The statement that the signature defines the key is to be understood with this restriction: actually the signature defines one pair of keys, one major, one minor. In order to tell which of the two is the key of a piece we must hear or see a little of the music itself.

Another distinguished pair of keys is the major and the minor

that have their *center* in common, as, for instance, C major and C minor. These two keys have of course different seven tone sets, different signatures (C minor replaces the tones e, a, and b of C major, $\hat{3}$, $\hat{6}$, and $\hat{7}$, by e flat, a flat, and b flat). But the sameness of center ties them together so closely that a change from one to the other, involving a change of mode only, is often not felt as an actual move. In practice they are frequently treated as two versions of the same key rather than two different keys.

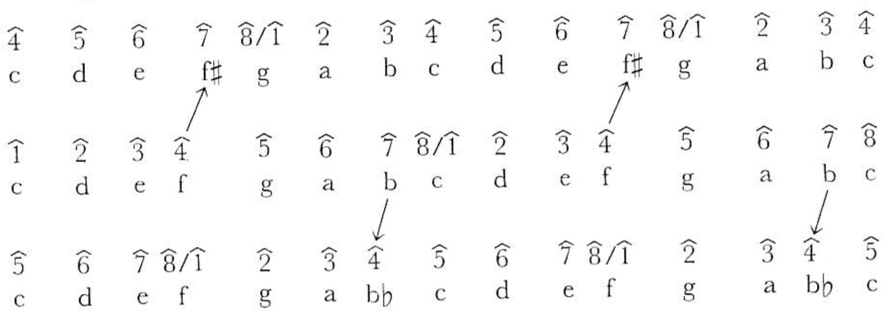
Most important of all key relations is that of two keys having all but one tone in common.

We approach the problem through the question: Are there any two such keys, two keys that have six tones in common? We state the question in terms of an arrangement of points in space: Given a series of points arranged in the pattern of the diatonic order, is it possible to change the position of just one point (including of course its "octave replicas") and get the same pattern again? We number the points in a "major scale" succession:

1 2 3 4 5 6 7 8

Using the trial and error method, we find easily that any shift in the position of points 2, 5, or 6 would immediately destroy the pattern by creating an oversize distance on the side opposite the shift. For the same reason, $\hat{3}$ or $\hat{7}$ cannot be shifted to the right, nor 1 or 4 to the left; but 3 cannot be moved to the left either, nor 1 (8) to the right, because this would produce four consecutive 'whole tones' on the opposite side of the move. Only two possibilities remain, the shift of 4 to the right and of 7 to the left. The following diagram shows that either of these shifts actually produces the desired result: after the shift the pattern reappears, moved into another position. 4 after the shift is 7 in the new position; 7 after the shift is 4 in the new position. One move is the reverse of the other, the two possibilities are at the bottom only one

Substituting tones for points, extending the pattern over two octaves, and starting from the C major scale as the middle, or zero, position, we get the following picture:



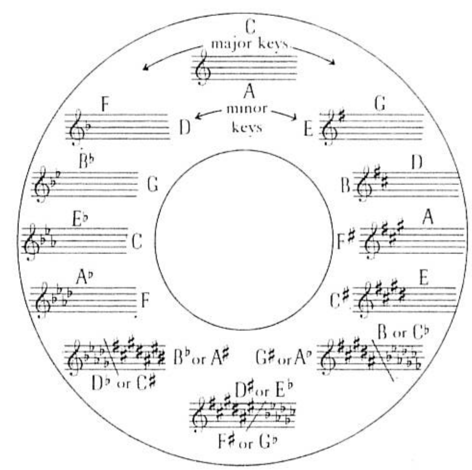
This diagram makes it plain that there are two keys, major mode, that have six tones common with C major, one on either side, as it were. It also shows us which keys they are. One is the result of the raising of f, $\hat{4}$ in C major, to f sharp, $\hat{7}$ in the new context; $\hat{8}$ is g, the new key is G major (signature: one sharp,). The tone g was $\hat{5}$ in C major: $\hat{5}$ has become $\hat{1}$. The other key results from the change of b, $\hat{7}$ in C major, to b flat, $\hat{4}$ in the new context. The key is F major (signature: one flat,). In F major the tone c is $\hat{5}$: $\hat{1}$ has become $\hat{5}$.

It is clear that the process can be continued with the same result in either direction. There are again two keys that have six tones in common with G major. One of them is C major. The other will be the result of G major's $\hat{4}$, the tone c, being raised to c sharp, which will be $\hat{7}$ in the new context, making d = $\hat{8}$. The key is D major (signature: one more sharp, two altogether,). In the same way there are two keys that have six tones in common with F major. One of them is C major. The other will result from F major's $\hat{7}$, e, being lowered to e flat, the new $\hat{4}$. $\hat{1}$ will be b flat (the key: B flat major; signature: one more flat, two of them altogether, . And so on. Every new step will add one more sharp or flat to the signature.

Every step is a transformation of $\hat{5}$ into $\hat{1}$ or $\hat{1}$ into $\hat{5}$. The same kind of relation exists of course between the minor keys. The only difference is in the numbers of the two tones whose change effects the key changes: in minor they are not $\hat{4}$ and $\hat{7}$ but $\hat{6}$ and $\hat{2}$.

If we call the pitch distance between the tones $\hat{1}$ and $\hat{5}$ a *fifth*, we can formulate the following law of key relations: Apart from the pair of relatives, those keys are closely related whose centers are a fifth distant of each other. This binds two keys to any given key in close relationship, one whose center lies a fifth above, the other with its center a fifth below, that of the given key—as in . In the chain of these relations every step "upward" from a given key will be expressed in the signature by the addition of one sharp or the taking away of one flat; every step "downward" by the addition of one flat or the taking away of one sharp. This explains the arrangement of sharps and flats in the signature, and also why no diatonic signature can contain sharps *and* flats: on the way from sharps to flats—and vice versa—the zero point of no sharps no flats—C major or A minor—must be passed.

The following is the so-called *Circle of Fifths*, the systematic tabulation, with their signatures, of all the keys of our music:



Three Types of Chromatic Tones

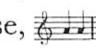
So far we have considered chromatic tones as a means of changing the center. This is not their only function, however; nor is the change of center always a necessary consequence of the appearance of chromatic tones. It depends entirely upon the way the tones move whether or not chromatic tones and chromatic steps affect the center. Here is an example where they do not:

This is the theme of the D minor fugue from Bach's *Well-Tempered Clavichord, II.* The line starts from $\hat{1}$, ascends in a kind of rolling motion to $\hat{5}$, then jumps from $\hat{5}$ directly to $\hat{8}$, without touching any intervening tones. On $\hat{8}$ it turns around and descends again, through $\hat{5}$ to $\hat{1}$. It is the descending stretch between $\hat{8}$ and $\hat{5}$ that catches our interest. We hear a succession of chromatic steps. They do not change the audible direction of the movement towards the tone d—d remains the goal, $\hat{1}$. The chromatic tones have had no effect on the center. What are they needed for, then?

They allow more moves to be squeezed into a given pitch distance than the diatonic pattern would provide! We need only hear the theme with a diatonic  replacing the chromatic  to realize the gain. It seems that the rolling motion from $\hat{1}$ up to $\hat{5}$, followed by the skip $\hat{5}$–$\hat{8}$, accumulates an impulse that the diatonic $\hat{8}$–$\hat{7}$–$\hat{6}$–$\hat{5}$ would allow to peter out much too quickly and ineffectually; in the chromatic sequence, on the other hand, the movement seems almost to force its way against a counter-pressure that constrains the tones to take smaller steps than they normally would. Thus, these chromatic tones do not affect the center, but they greatly affect the character of the *movement.*

Since this example is in D minor one could argue that the tones involved in the chromatic motion were not chromatic tones in a strict sense; the chromatic motion resulted from the successive touching of the two alternative positions of the tones $\hat{7}$ and $\hat{6}$—the sharpened first, then the diatonic—so that actually no tones foreign to the key were introduced. However, the story is the same in major, as shown by the following melody from the minuet of Mozart's String Quartet in G major, K. 387:

The same tone g which is $\hat{1}$ at the beginning is heard as $\hat{1}$ immediately after the chromatic sequence, which therefore cannot have affected the center.

Another typical use of chromatic tones occurs in this theme from the finale of Mozart's *Haffner* Symphony, K. 335: . The g sharp is certainly a chromatic tone in D major, the key of the piece; however, the dynamic quality of the  that concludes each of the two short phrases is exactly the same after the second as after the first, the chromatic tone does not disturb the center at all. It is a sort of private excursion of the tone a, $\hat{5}$: instead of merely repeating that tone as in the first phrase, , the movement allows it a margin of freedom just big enough to accomplish a swift away-and-back move, touching the tone that *would be* its $\hat{7}$ if it were to become $\hat{1}$, the tone g sharp. Music is full of such would-be $\hat{7}$–$\hat{8}$ moves involving chromatic tones, which never have a chance to transform the tone at which they aim into a center; they leave the dynamic quality of that tone unaffected —as in our example the move from g sharp to a did not alter the dynamic quality of a = $\hat{5}$. Yet whenever they appear they attract the attention to that point; they distribute a series of lights of varying intensity over the line that ventures into such excursions, producing all sorts of subordinate tensions and releases—as for instance in this melody from Mozart's E flat major String Quartet, K. 428:

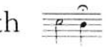
This kind of indirect language—the replacing of the diatonic tones actually meant by the chromatic tones most sharply pointing at them—which is used with light touch here, can be intensified to produce the highest degree of emphasis, the strongest inner accents of which tonal expression is capable.

The three types of chromatic tones, those that change the center, those that change the character of the motion, and those that stress individual tones, are not strictly separated in actual music. All sorts of mixtures and combinations of these types occur all the time: strong emphasis on a particular tone may be carried to the point of suggesting for a passing moment that this tone has become $\hat{1}$; chromatic motion may include a stressing of particular tones; and so on.

Move of Center Without Chromatic Tones

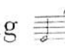
Ordinarily a shift of center carries with it the appearance of chromatic tones; but just as on the one hand chromatic tones appear that do not change the center, so on the other a change of center may occur without the appearance of chromatic tones. Merely by the way they move, without expressedly breaking the pattern, the tones can communicate to the ear that their center of attraction has shifted to another place. The following hymn tune shows how this happens.

The key is G minor (the f sharp in the first measure is not a chromatic tone but a tone which belongs: raised $\hat{7}$). After the

initial $\hat{8}$–$\hat{7}$–$\hat{5}$–$\hat{8}$ motion that fixes the center, the line ascends to c, $\hat{4}$, and turns back again, coming to a halt on the tone a, $\hat{2}$. The next phrase begins with a move away from $\hat{1}$, 2–3; when the phrase comes to a conclusion in measure 6, with  , this last tone, b flat, is clearly heard as $\hat{1}$. So in the course of this phrase the center has changed place. How?

The tones involved in this phrase form a pitch pattern that can be interpreted in two ways:

	a	b♭	c	d	e♭
G minor	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$
B♭ major	$\hat{7}$	$\hat{8}/\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$

As the movement fails to reassert g and instead turns repeatedly to the tone b flat, the ear somehow loses sight of the original center and accepts b flat as $\hat{1}$. It is as if a slight loosening of the grip of center g on the movement had given the other tone the chance to establish itself in the position of center. Only for a short time, though. The following phrase, after moving away from b flat, $\hat{8}$–$\hat{7}$–$\hat{6}$–$\hat{5}$, in its turn fails to reaffirm the new center; it seems as if the original center, awakened from its inactivity, would not allow this to happen. Very delicately it draws the movement back into the original orbit, first, by leaving the situation somewhat in doubt at the phrase ending  (second measure from the last)—is this $\hat{4}$–$\hat{3}$ or $\hat{6}$–$\hat{5}$?—then, by lifting the tone d up an octave and attracting the motion definitively toward itself: so that d *was* $\hat{5}$, and the end is $\hat{5}$–$\hat{4}$–$\hat{3}$–$\hat{2}$–$\hat{1}$.

The shift in this case involved G minor and B flat major, the relatives that have all seven diatonic tones in common. Another tune shows a similar development, a shift from e flat = $\hat{1}$ to b flat = $\hat{1}$ and back again, all in major. The diatonic sets involved have six tones in common.

The original center loses its hold on the motion when in measure 6 the melody fails to move from $\hat{7}$, d, to $\hat{8}$, e flat, , and instead turns around and comes to a halt on b flat, . The return to the original center is accomplished in one moment, with the appearance of the tone a flat in the move that follows immediately upon the assertion of b flat as $\hat{1}$. Had the melody wished to maintain the new center in power, it would have moved .

It is obvious that the fewer tones two seven tone sets have in common, the less likely it will be for the shift of center to occur without chromatic tones.

Intervals

One tone is not yet music. One might say it is only a promise of music. The promise is fulfilled, and music comes into being, only when tone follows upon tone. Strictly speaking, therefore, the basic elements of music are not the individual tones but the individual tone-to-tone moves.

Each one of these moves spans a certain pitch distance. The pitch distance between two tones is called an *interval.* If the basic elements of a melody are the individual moves, melody is a succession of intervals rather than of tones.

There are two types of moves according to the intervals spanned; the following theme from Bach's Two-part Invention in F major shows them neatly side by side: . In the second half of this theme the motion, upward or downward, proceeds throughout from tone to neighboring tone on the scale. This is called *stepwise* motion. In the first half, on the contrary, every move skips one or more intervening scale tones. Compared to the homogeneity of the stepwise motion the motion by skips shows great variety. It is clear that the combination of stepwise and skipwise motion will be a major factor in deter-

mining the profile, the personal character of a melody. In the hymn tunes we have quoted, the motion was mostly stepwise. In a way, stepwise motion can be considered normal motion, in the sense that it involves the least effort in the move from tone to tone; while every skip goes beyond the norm in that it expresses a greater effort by taking us to a more distant tone more rapidly than the normal succession of intervening steps would permit.

Intervals are identified and named according to the number of successive scale tones that go into the pitch distance. The interval between any two successive scale tones is called a *second* (, or , or , and so on); the interval between any tone of the scale and the third tone from it is called a *third* (, or , or , etc.). Next comes the *fourth,* etc., then the *fifth,* etc., the *sixth,* etc., the *seventh,* etc., the *octave,* . Intervals larger than the octave—the *ninth,* the *tenth,* and so on—are rarely used, except in some 20th century music.

Any two intervals that taken together make up an octave are called *complementary.* Such pairs are:

second	+ seventh	= octave
third	+ sixth	= octave
fourth	+ fifth	= octave

These relationships are important because they give the tones the freedom to perform, for instance, the move $\hat{1}$–$\hat{2}$ either as a second up or as a seventh down , the move $\hat{5}$–$\hat{1}$ either as a fourth up (where we call it $\hat{5}$–$\hat{8}$) or as a fifth down . (In current terminology this relationship of intervals is called *inversion.* This creates confusion, as the same term is used also for another, entirely different interval relation: namely, between, for example, a second up and a second down, a third up and a third down.)

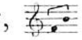
The complementary interval of the octave is the *prime:* an interval by name only, as here the pitch distance between the two tones is zero.

We realize that the number of successive scale steps is not a very reliable means of measuring pitch distances since the distance between successive scale tones is not always the same; sometimes it is a whole tone, sometimes a half tone. Accordingly intervals called by the same name will not always measure the same pitch distance. In the major scale, for instance, the second between the 1st and 2nd, and between the 2nd and 3rd tones measures one whole tone; but the second between the 3rd and 4th tones measures one half tone. Consequently the third between the 1st and 3rd tones will measure two whole tones, the third between the 2nd and 4th tones one whole plus one half tone. A corresponding difference will appear in the complementary intervals: the seventh that complements the larger second will be smaller than the seventh that complements the smaller second; the same will be true for the sixth.

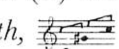
We have therefore to distinguish between two kinds of seconds, thirds, sixths, and sevenths, which are called *major* and *minor* second, third, sixth, and seventh. The difference in every case is one half tone. ("Major" and "minor" in this context must not be confused with "major" and "minor" as a distinction of mode; minor intervals occur in the major mode—e.g. the minor second between the 3rd and 4th tones, the minor third between the 2nd and 4th tones—and vice versa.)

The situation is somewhat different for the remaining pair of intervals, the fourth and the fifth. As we can see from the scale pattern, all the fourths, except one, are alike—each measuring two whole plus one half tones; so all the fifths, except one, must be alike, measuring three whole plus one half tones each. These fourths and fifths are called *perfect* and distinguished from the two exceptions, the *augmented* fourth which is one half tone too large, and the *diminished* fifth which is one half tone too small.

The striking contrast between the "good" sound of the perfect and the "bad" sound of the augmented and diminished intervals will justify the use of this special terminology.

The augmented fourth occurs between the tones $\hat{4}$ and $\hat{7}$ of the major scale (for instance, in C major, ); it measures three whole tones. The diminished fifth occurs between the tones $\hat{7}$ and $\hat{4}$ (above), ; it measures two whole and two half tones. The two pitch distances are the same; thus, these intervals may be said to divide the octave exactly in half. We have noted once before, in the case of the chromatic scale, that the quantitatively most satisfactory distribution produces the most unsatisfactory musical result.

The octaves are all alike, all *perfect*.

Two pairs of odd-sized intervals are occasionally created as a consequence of the raising of $\hat{7}$ in minor. They are: (1) the *diminished seventh* (e.g. in A minor, ) with its complement, the *augmented second*, ; and (2) the *augmented fifth* with its complement, the *diminished fourth*, .

Chromatic tones may produce other distortions of intervals, like augmented and diminished octaves; and so on.

Tone and Number

These intervals of the diatonic order possess an extraordinary property, which has caused amazement and wonder ever since it was discovered by the ancient Greeks, reputedly by Pythagoras.

We take two strings, stretched with the same tension over a sounding board, the second more than twice as long as the first. If plucked, string 2 will produce a much lower tone than string 1.

It is clear that in order to produce the higher tone on string 2 I must reduce the length of the vibrating stretch to exactly